Non-Gaussianities from isocurvature modes

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Abstract. This contribution discusses isocurvature modes, in particular the non-Gaussianities of local type generated by these modes. Since the isocurvature transfer functions differ from the adiabatic one, the coexistence of a primordial isocurvature mode with the usual adiabatic mode leads to a rich structure of the angular bispectrum, which can be decomposed into six elementary bispectra. Future analysis of the CMB data will enable to measure their relative weights, or at least constrain them. Non-Gaussianity thus provides a new window on isocurvature modes. This is particularly relevant for some scenarios, such as those presented here, which generate isocurvature modes whose contribution in the power spectrum is suppressed, as required by present data, but whose contribution in the non-Gaussianities could be dominant and measurable.

1. Introduction

Inflation is currently the best candidate to explain the generation of primordial perturbations (see e.g. [1] for a recent pedagogical introduction), but current observations cannot point to a specific scenario. The hope is thus that future data will enable us to discriminate between various models. In this respect, an important distinction is between single-field and multiple-field models. The detection of even a tiny fraction of isocurvature mode in the cosmological data would rule out single-field inflation, which predicts only adiabatic perturbations. Another signature that could distinguish multiple-field models from single-field models is a detectable primordial non-Gaussianity of the local type. Similarly to isocurvature modes, a detection of local primordial non-Gaussianity would rule out all inflation models based on a single scalar field, since they generate only unobservably small local non-Gaussianities.

This contribution, based on the recent works [2, 3, 4, 5], discusses how isocurvature modes could affect non-Gaussianities and their specific signature in the Cosmic Microwave Background (CMB) fluctuations. In particular, it is shown how the bispectrum generated by the adiabatic mode together with one isocurvature mode leads to a total angular bispectrum which can be decomposed into six distinct components: the usual purely adiabatic bispectrum, a purely isocurvature bispectrum, and four other bispectra that arise from the possible correlations between the adiabatic and isocurvature mode. Because these six bispectra have different shapes in angular space, their amplitude can in principle be measured in the CMB data.

This analysis opens a new window on isocurvature modes, especially important to test models where the isocurvature contribution is suppressed in the power spectrum but not in the non-Gaussianities. Examples of such models are presented at the end of this contribution.

2. Isocurvature modes

At the time of last scattering, the main components in the Universe are the CDM (c), the baryons (b), the photons (γ) and the neutrinos (ν). All these components are characterized by their individual energy density contrasts δ_i. The most common type of perturbation is the adiabatic mode, characterized by the condition

$$\delta_c = \delta_b = \frac{3}{4}\delta_\nu = \frac{3}{4}\delta_\gamma \,, \tag{1}$$

which means that the number of photons (or neutrinos, or CDM particles) per baryon does not fluctuate. Assuming adiabatic initial conditions is natural if all particles have been created by the decay of a single degree of freedom, such as a single inflaton, and, so far, the CMB data are fully compatible with purely adiabatic perturbations.

However, other types of perturbations can be included in a more general framework. In addition to the adiabatic mode, one can consider four distinct isocurvature modes [6]: the CDM isocurvature mode, the baryon isocurvature mode, the neutrino density isocurvature mode and the neutrino velocity isocurvature mode. The first three isocurvature modes are characterized by

$$S_X = \frac{1}{1+w_X}\delta_X - \frac{3}{4}\delta_\gamma, \qquad X = \{c, b, \nu d\} \tag{2}$$

with $w_X = P_X/\rho_X$. As for the neutrino velocity isocurvature mode, it is characterized by a non vanishing "initial velocity", compensated by the velocity of the photon-baryon plasma so that the total momentum density is cancelled, while the energy densities satisfy (1).

In the following, these five modes will be denoted collectively as X^I. In the context of inflation, a necessary, although not sufficient, condition for at least one of these isocurvature modes to be produced is that several light degrees of freedom exist during inflation. Moreover, since the adiabatic and isocurvature modes can be related in various ways to these degrees of freedom during inflation, one can envisage the existence of correlations between these modes [7].

The various modes lead to *different* predictions for the CMB temperature and polarization. Let us consider for instance the temperature anisotropies, which can be decomposed into spherical harmonics:

$$\frac{\Delta T}{T} = \sum_{lm} a_{lm} Y_{lm} \,. \tag{3}$$

At linear order, the multipole coefficients a_{lm} are related to a general primordial perturbation consisting of the superposition of several modes, via the expression

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left(\sum_I X^I(\mathbf{k})\, g_l^I(k) \right) Y_{lm}^*(\hat{\mathbf{k}}) \,, \tag{4}$$

where $g_l^I(k)$ denotes the transfer function associated with the mode X^I. As a result, the total angular power spectrum is given by

$$C_l = \langle a_{lm} a_{lm}^* \rangle = \sum_{I,J} \frac{2}{\pi} \int_0^\infty dk\, k^2 g_l^I(k) g_l^J(k) P_{IJ}(k) \,, \tag{5}$$

where the primordial power spectra $P_{IJ}(k)$ are defined by

$$\langle X^I(\mathbf{k}_1) X^J(\mathbf{k}_2) \rangle \equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) P_{IJ}(k_1) \,. \tag{6}$$

Since the various transfer functions are *different*, this leads to different predictions for the CMB angular power spectrum. This is illustrated in Fig. 1 (left panel), where the angular power

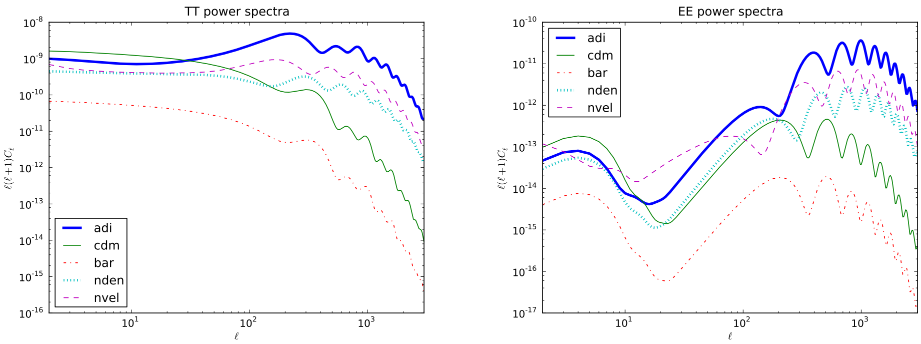

Figure 1. Angular power spectra (multiplied by $l(l+1)$) for the temperature (left) and polarization (right) obtained from purely adiabatic or purely isocurvature initial conditions. The amplitude and spectral index of the primordial power spectrum, as well as the cosmological parameters, on which the transfer functions depend, correspond to the WMAP7-only best-fit parameters.

spectra produced separately by the various modes are plotted, assuming the same primordial power spectrum. The only exception are the CDM and baryon isocurvature modes which give exactly the same pattern, up to the rescaling $S_b = (\Omega_b/\Omega_c)\, S_c$ where Ω_b and Ω_c denote, as usual, the present energy density fractions, respectively for baryons and CDM.

We infer from CMB observations that the "primordial" perturbation is mainly of the adiabatic type. However, this does not preclude the presence, in addition to the adiabatic mode, of an isocurvature component, with a smaller amplitude. Precise measurement of the CMB fluctuations could lead to a detection of such an extra component, or at least put constraints on its amplitude. For example, constraints on the CDM isocurvature to adiabatic ratio,

$$\alpha = \frac{\mathcal{P}_{S_c}}{\mathcal{P}_\zeta}, \tag{7}$$

based on the WMAP7+BAO+SN data, have been published for the uncorrelated and fully correlated cases (the impact of isocurvature perturbations on the observable power spectrum indeed depends on the correlation between adiabatic and isocurvature perturbations, as illustrated in [8]). In terms of the parameter $a \equiv \alpha/(1+\alpha)$, the limits given in [9] are

$$a_0 < 0.064 \quad (95\%\mathrm{CL}), \qquad a_1 < 0.0037 \quad (95\%\mathrm{CL})\,, \tag{8}$$

respectively for the uncorrelated case and for the fully correlated case.

3. Generalized angular bispectra

Let us now turn to the CMB non-Gaussianities. The angular bispectrum corresponds to the three-point function of the multipole coefficients:

$$B^{m_1 m_2 m_3}_{l_1 l_2 l_3} \equiv \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle\,. \tag{9}$$

Substituting the expression (4) into the angular bispectrum, one finds

$$B^{m_1 m_2 m_3}_{l_1 l_2 l_3} = \mathcal{G}^{m_1 m_2 m_3}_{l_1 l_2 l_3}\, b_{l_1 l_2 l_3}\,, \tag{10}$$

where the first, purely geometrical, factor is the Gaunt integral

$$\mathcal{G}^{m_1 m_2 m_3}_{l_1 l_2 l_3} \equiv \int d^2\hat{\mathbf{n}}\, Y_{l_1 m_1}(\hat{\mathbf{n}})\, Y_{l_2 m_2}(\hat{\mathbf{n}})\, Y_{l_3 m_3}(\hat{\mathbf{n}})\,, \tag{11}$$

while the second factor, usually called the *reduced* bispectrum,

$$\begin{aligned} b_{l_1 l_2 l_3} &= \sum_{I,J,K} \left(\frac{2}{\pi}\right)^3 \int \left(\prod_{i=1}^{3} k_i^2 dk_i\right) \, g_{l_1}^I(k_1) g_{l_2}^J(k_2) g_{l_3}^K(k_3) \\ & \quad \times B^{IJK}(k_1,k_2,k_3) \int_0^\infty r^2 dr j_{l_1}(k_1 r) j_{l_2}(k_2 r) j_{l_3}(k_3 r) \,, \end{aligned} \tag{12}$$

depends on the bispectra of the primordial X^I:

$$\langle X^I(\mathbf{k}_1) X^J(\mathbf{k}_2) X^K(\mathbf{k}_3)\rangle \equiv (2\pi)^3 \delta(\Sigma_i \mathbf{k}_i) B^{IJK}(k_1,k_2,k_3) \,. \tag{13}$$

We now need to specify the primordial bispectra B^{IJK}. For purely adiabatic perturbations, the local bispectrum is expressed as the square of the power spectrum (symmetrized over k_1, k_2 and k_3). Here, we consider the generalization

$$B^{IJK}(k_1,k_2,k_3) = \tilde{f}_{\rm NL}^{I,JK} P_\zeta(k_2) P_\zeta(k_3) + \tilde{f}_{\rm NL}^{J,KI} P_\zeta(k_1) P_\zeta(k_3) + \tilde{f}_{\rm NL}^{K,IJ} P_\zeta(k_1) P_\zeta(k_2) \,, \tag{14}$$

where the coefficients $\tilde{f}_{\rm NL}^{I,JK}$ satisfy the condition

$$\tilde{f}_{\rm NL}^{I,JK} = \tilde{f}_{\rm NL}^{I,KJ} \,. \tag{15}$$

The above expression is the natural outcome of a generic model of multiple-field inflation. Indeed, allowing for several light degrees of freedom during inflation, one can relate, in a very generic way, the "primordial" perturbations X^I (defined during the standard radiation era) to the fluctuations of light primordial fields ϕ^a, generated at Hubble crossing during inflation, so that one can write, up to second order,

$$X^I = N_a^I \, \delta\phi^a + \frac{1}{2} N_{ab}^I \, \delta\phi^a \delta\phi^b + \ldots \tag{16}$$

where the $\delta\phi^a$ can usually be treated as independent quasi-Gaussian fluctuations, i.e.

$$\langle \delta\phi^a(\mathbf{k}) \, \delta\phi^b(\mathbf{k}')\rangle = (2\pi)^3 \, \delta^{ab} P_{\delta\phi}(k) \, \delta(\mathbf{k}+\mathbf{k}') \,, \qquad P_{\delta\phi}(k) = 2\pi^2 k^{-3} \left(\frac{H_*}{2\pi}\right)^2 \,, \tag{17}$$

where a star denotes Hubble crossing time. The relation (16) is very general, and all the details of the inflationary model are embodied by the coefficients N_a^I and N_{ab}^I. Substituting (16) into (13) and using Wick's theorem, one finds that the bispectra B_{IJK} can be expressed in the form

$$B^{IJK}(k_1,k_2,k_3) = \lambda^{I,JK} P_{\delta\phi}(k_2) P_{\delta\phi}(k_3) + \lambda^{J,KI} P_{\delta\phi}(k_1) P_{\delta\phi}(k_3) + \lambda^{K,IJ} P_{\delta\phi}(k_1) P_{\delta\phi}(k_2) \,, \tag{18}$$

with the coefficients

$$\lambda^{I,JK} \equiv \delta^{ac} \delta^{bd} N_{ab}^I N_c^J N_d^K \tag{19}$$

(the summation over scalar field indices a, b, c and d is implicit), which are symmetric under the interchange of the last two indices, by construction. Since the adiabatic power spectrum is given by

$$P_\zeta = (\delta^{ab} N_a^\zeta N_b^\zeta) P_{\delta\phi} \equiv A P_{\delta\phi}, \tag{20}$$

one obtains finally (14) with

$$\tilde{f}_{\rm NL}^{I,JK} = \lambda_{NL}^{I,JK} / A^2 \,, \tag{21}$$

where it is implicitly assumed that the coefficients N_a^I are weakly time dependent so that the scale dependence of A^2 can be neglected.

After substitution of (14) into (12), the reduced bispectrum can finally be written as

$$b_{l_1l_2l_3} = \sum_{I,J,K} \tilde{f}_{\rm NL}^{I,JK} b_{l_1l_2l_3}^{I,JK}, \tag{22}$$

where each contribution is of the form[1]

$$b_{l_1l_2l_3}^{I,JK} = 3\int_0^\infty r^2 dr\, \alpha_{(l_1}^I(r)\beta_{l_2}^J(r)\beta_{l_3)}^K(r), \tag{23}$$

with

$$\alpha_l^I(r) \equiv \frac{2}{\pi}\int k^2 dk\, j_l(kr)\, g_l^I(k), \qquad \beta_l^I(r) \equiv \frac{2}{\pi}\int k^2 dk\, j_l(kr)\, g_l^I(k)\, P_\zeta(k)\,. \tag{24}$$

4. Observational prospects

For simplicity, we assume that the primordial perturbation is the combination of the dominant adiabatic mode with a *single* isocurvature mode. In this case, the total bispectrum is characterized by *six* parameters, which we now denote $\tilde{f}^{(i)}$,

$$\begin{aligned} b_{l_1l_2l_3} &= \tilde{f}^{\zeta,\zeta\zeta}\, b_{l_1l_2l_3}^{\zeta,\zeta\zeta} + 2\tilde{f}^{\zeta,\zeta S}\, b_{l_1l_2l_3}^{\zeta,\zeta S} + \tilde{f}^{\zeta,SS}\, b_{l_1l_2l_3}^{\zeta,SS} + \tilde{f}^{S,\zeta\zeta}\, b_{l_1l_2l_3}^{S,\zeta\zeta} + 2\tilde{f}^{S,\zeta S}\, b_{l_1l_2l_3}^{S,\zeta S} + \tilde{f}^{S,SS}\, b_{l_1l_2l_3}^{S,SS} \\ &= \sum_{(i)} \tilde{f}^{(i)} b_{l_1l_2l_3}^{(i)}\,, \end{aligned} \tag{25}$$

where the index i varies between 1 to 6, following the order indicated in the upper line. Note that, because of the factor 2 in front of $\tilde{f}^{\zeta,\zeta S}$ and $\tilde{f}^{S,\zeta S}$, we define $b_{l_1l_2l_3}^{(2)} \equiv 2b_{l_1l_2l_3}^{\zeta,\zeta S}$ and $b_{l_1l_2l_3}^{(5)} \equiv 2b_{l_1l_2l_3}^{S,\zeta S}$ whereas there is no such factor 2 for the other terms.

To estimate these six parameters, given some data set, the usual procedure is to minimize

$$\chi^2 = \left\langle (B^{obs} - \sum_i \tilde{f}^{(i)} B^{(i)}), (B^{obs} - \sum_i \tilde{f}^{(i)} B^{(i)}) \right\rangle, \tag{26}$$

where B is a short notation for the angle-averaged bispectrum

$$B_{l_1l_2l_3} \equiv \sum_{m_1,m_2,m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{l_1l_2l_3}^{m_1m_2m_3}\,. \tag{27}$$

For an ideal experiment (no noise and no effects due to the beam size) without polarization, the scalar product in (26) is defined by

$$\langle B, B'\rangle \equiv \sum_{l_1\le l_2\le l_3} \frac{B_{l_1l_2l_3}B'_{l_1l_2l_3}}{\sigma_{l_1l_2l_3}^2}. \tag{28}$$

with the variance

$$\sigma_{l_1l_2l_3}^2 \equiv \langle B_{l_1l_2l_3}^2\rangle - \langle B_{l_1l_2l_3}\rangle^2 \approx (1+\delta_{l_1l_2}+\delta_{l_2l_3}+\delta_{l_3l_1}+2\,\delta_{l_1l_2}\delta_{l_2l_3})\, C_{l_1}C_{l_2}C_{l_3} \tag{29}$$

in the approximation of weak non-Gaussianity.

The best estimates for the parameters are thus obtained by solving

$$\sum_j \langle B^{(i)}, B^{(j)}\rangle \tilde{f}^{(j)} = \langle B^{(i)}, B^{obs}\rangle\,, \tag{30}$$

[1] We use the standard notation: $(l_1l_2l_3) \equiv [l_1l_2l_3 + 5\,\text{perms}]/3!$.

while the statistical error on the parameters is deduced from the second-order derivatives of χ^2, which define the Fisher matrix, given here by

$$F_{ij} \equiv \langle B^{(i)}, B^{(j)} \rangle. \tag{31}$$

For a real experiment, and if E-polarization is included as well, the above equations remain valid, except that the definition of the scalar product has to be replaced by a more complicated expression (see [5] for details).

For each of the four isocurvature modes, the corresponding Fisher matrix has been computed numerically, including the polarization, for the noise characteristics of the Planck satellite in [4, 5]. The error on the parameters $\tilde{f}^i$ can then be deduced from the components of the Fisher matrix, via the expression

$$\Delta \tilde{f}^i = \sqrt{(F^{-1})_{ii}} \tag{32}$$

For the various cases, we have obtained

$$\begin{aligned}
\Delta \tilde{f}^i &= \{9.6, 7.1, 160, 150, 180, 140\} \qquad \text{(CDM isocurvature)} && (33)\\
\Delta \tilde{f}^i &= \{9.6, 35, 4000, 720, 4300, 16600\} \qquad \text{(baryon isocurvature)} && (34)\\
\Delta \tilde{f}^i &= \{28, 36, 190, 150, 240, 320\} \qquad \text{(neutrino density isocurvature)} && (35)\\
\Delta \tilde{f}^i &= \{25, 22, 85, 81, 77, 71\} \qquad \text{(neutrino velocity isocurvature)} && (36)
\end{aligned}$$

As one can see, the error on the first two parameters is much smaller than the last four parameters in the CDM isocurvature case. An explanation for this result is given in [5].

5. Illustrative example

To illustrate the previous results, which are model-independent, it is instructive to consider a simple class of models based on the presence of a spectator light scalar field during inflation, dubbed curvaton. This curvaton acquires nearly scale-invariant fluctuations during inflation and, later, behaves as a pressureless fluid when it oscillates at the bottom of its potential, before decaying.

Here, we allow the curvaton σ to decay into both radiation and CDM with the respective branching ratios γ_r and γ_c. Since, in general, CDM can already be present before the decay, we define the fraction of CDM created by the decay as $f_c \equiv \gamma_c \Omega_\sigma/(\Omega_c + \gamma_c \Omega_\sigma)$, where the Ω's represent the relative abundances just before the decay.

As shown in [2], the "primordial" adiabatic and isocurvature perturbations, i.e. defined after the curvaton decay, can be written in the form (16), with

$$N_\sigma^\zeta = \frac{2r}{3\sigma_*}, \qquad N_{\sigma\sigma}^\zeta = \frac{2r}{3\sigma_*^2}, \tag{37}$$

$$N_\sigma^S = \frac{2}{\sigma_*}(f_c - r), \qquad N_{\sigma\sigma}^S = \frac{2}{\sigma_*^2}\left[f_c(1-2f_c) - r\right], \tag{38}$$

where $r \equiv 3\,\gamma_r\,\Omega_\sigma/[(4-\Omega_\sigma)(1-(1-\gamma_r)\Omega_\sigma)]$ is assumed to be small, since significant non-Gaussianities arise only if $r \ll 1$.

Let us first discuss linear perturbations. It is useful to introduce the curvaton contribution to the total adiabatic power spectrum $\Xi \equiv (N_\sigma^\zeta)^2/[(N_\phi^\zeta)^2 + (N_\sigma^\zeta)^2]$, where $N_\phi^\zeta = H/\dot{\phi}$ is associated with the inflaton fluctuation, and $N_\phi^S = 0$. Ξ is directly related to the correlation $\mathcal{C} \equiv P_{\zeta,S}/\sqrt{P_S P_\zeta} = \sqrt{\Xi}\,\mathrm{sgn}(f_c - r)$. The isocurvature-adiabatic ratio, given by

$$\alpha \equiv \frac{P_S}{P_\zeta} = \frac{(N_\sigma^S)^2}{(N_\phi^\zeta)^2 + (N_\sigma^\zeta)^2} = 9\left(1 - \frac{f_c}{r}\right)^2 \Xi\,, \tag{39}$$

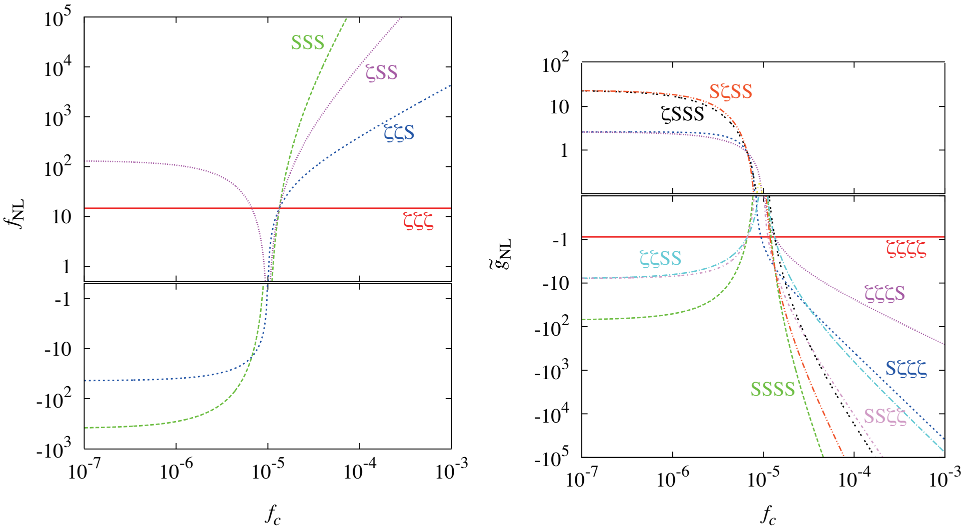

Figure 2. Plots of the coefficients $\tilde{f}_{\rm NL}^{I,JK}$ (left panel) and of $\tilde{g}_{\rm NL}^{IJKL}$ (right panel) as functions of f_c. I, J, K, L are specified in the figure for each line. Other parameters are fixed as $\xi = 1, \lambda = 10^{-3}$ and $r = 10^{-5}$.

is constrained by CMB observations [9] to be small which requires either $f_c \simeq r$ or $\Xi \ll 1$.

Let us now turn to non-Gaussianities. Using (37), one finds $\tilde{f}_{\rm NL}^{\zeta,\zeta\zeta} = 3\,\Xi^2/(2r)$. This is the dominant contribution in the regime $f_c \simeq r$, the other components being suppressed. We thus concentrate on the more interesting case $\Xi \ll 1$ to discuss the size of the various components in terms of f_c and r, considered as free parameters in our phenomenological approach.

In the regime $f_c \ll r \ll 1$, the purely adiabatic coefficient is the smallest one. The other ones are *enhanced* by powers of (-3) (since $N_\sigma^S/N_\sigma^\zeta = N_{\sigma\sigma}^S/N_{\sigma\sigma}^\zeta = -3$):

$$\tilde{f}_{\rm NL}^{I,JK} = (-3)^p \tilde{f}_{\rm NL}^{\zeta,\zeta\zeta}\,, \quad \tilde{f}_{\rm NL}^{\zeta,\zeta\zeta} = \frac{\alpha^2}{54r}\,, \tag{40}$$

where p is the number of "S" in the triplet $\{I, J, K\}$. In particular, the purely isocurvature coefficient is enhanced by a factor 27, but with the opposite sign: $\tilde{f}_{\rm NL}^{S,SS} = -\alpha^2/(2r)$. All coefficients can be significant if r is sufficiently smaller than α^2.

In the opposite regime $r \ll f_c \ll 1$, the purely adiabatic coefficient is, once again, the smallest one. All the coefficients are now positive and enhanced by factors $(3f_c/r)^p$, where p is again the number of "S" indices:

$$\tilde{f}_{\rm NL}^{I,JK} = \left(\frac{3f_c}{r}\right)^p \tilde{f}_{\rm NL}^{\zeta,\zeta\zeta}\,, \quad \tilde{f}_{\rm NL}^{\zeta,\zeta\zeta} = \frac{\alpha^2 r^3}{54 f_c^4}\,. \tag{41}$$

Note that the enhancement factor is much bigger than in the previous case (40). The purely isocurvature coefficient, which dominates, is $\tilde{f}_{\rm NL}^{S,SS} = \alpha^2/(2f_c)$ and can be large if f_c is sufficiently small, while the relative size of the other coefficients depends on the ratio r/f_c. The full dependence of the coefficients on the parameter f_c is illustrated in the left panel of Fig. 2.

A similar analysis for the trispectrum has been presented in [3]. The usual coefficients $\tau_{\rm NL}$ and $g_{\rm NL}$ that describe the local trispectrum are then generalized into, respectively, nine $\tau_{\rm NL}$-like coefficients and eight $g_{\rm NL}$-like coefficients:

$$\tau_{\rm NL}^{IJ,KL}\,, \qquad \tilde{g}_{\rm NL}^{I,JKL} \equiv \frac{54}{25} g_{\rm NL}^{I,JKL}\,. \tag{42}$$

The behaviour of the $g_{\rm NL}$-like coefficients as functions of f_c is plotted on the right panel of Fig. 2. The hierarchies are very similar to those observed for the bispectrum parameters.

In conclusion, the above results show that a small isocurvature fraction in the power spectrum is compatible with a dominantly isocurvature non-Gaussianity detectable by Planck (e.g. $\alpha \simeq 10^{-2}$ and $r \ll f_c \simeq 10^{-8}$ yields $\tilde{f}_{\rm NL}^{S,SS} \simeq 5 \times 10^3$). Of course, the relations (40) or (41), are specific to the models considered here and would be a priori different in other models. It is therefore important to try to measure these six coefficients *separately*, in order to obtain model-independent constraints from observations.

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